A model of a fishery with fish storage and variable price

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Modeling of fisheries systems

Figure – Vernon L. Smith

$$\begin{cases} \dfrac{dn}{dt} = f(n) - h(n,E), \\ \\ \dfrac{dE}{dt} = \phi\Big(ph(n,E) - cE\Big) \end{cases}$$

- Smith V.L. (1968). Economics of Production from Natural Resources. American Economic Review, 409-431
- Smith V.L. (1969). On models of commercial fishing. Journal of Political Economy, 77, 181-198

Fishery modelling

$$\begin{cases} \dfrac{dn}{dt} = f(n) - h(n,E), \\[2ex] \dfrac{dE}{dt} = \phi\Big(ph(n,E) - cE\Big) \end{cases}$$

Fishery modelling

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Fishery modelling

$$\begin{cases} \dfrac{dn}{dt} = f(n) - \boxed{h(n,E)} \\[2ex] \dfrac{dE}{dt} = \phi\Big(ph(n,E) - cE\Big) \end{cases}$$

Fishery modelling

$$\begin{cases} \dfrac{dn}{dt} = f(n) - h(n, E) \\ \\ \boxed{\dfrac{dE}{dt} = \phi\Big(ph(n, E) - cE\Big)} \end{cases}$$

Fishery modelling

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Fishery modelling

- $f(n) = rn(1 - \frac{n}{K})$

$$\begin{cases} \dfrac{dn}{dt} = \boxed{f(n)} - \boxed{h(n,E)} \\ \dfrac{dE}{dt} = \phi\Big(ph(n,E) - cE\Big) \end{cases}$$

- $h(n,E) = qnE$

Fishery modelling

- $f(n) = rn(1 - \frac{n}{K})$

$$\begin{cases} \dfrac{dn}{dt} = \boxed{f(n)} - \boxed{h(n,E)} \\ \\ \dfrac{dE}{dt} = \phi\Big(ph(n,E) - cE\Big) \end{cases}$$

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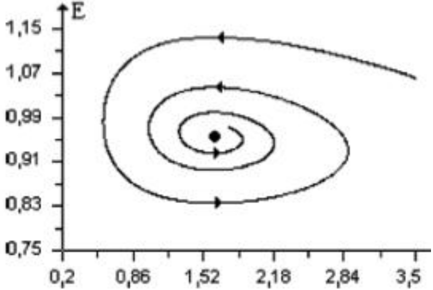
Fishery modelling

- $f(n) = rn(1 - \frac{n}{K})$
- $h(n, E) = qnE$

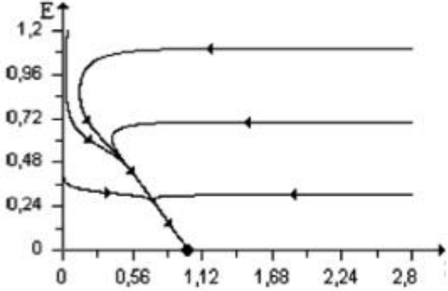
$$\begin{cases} \dfrac{dn}{dt} = rn(1 - \dfrac{n}{K}) - qnE \\[2ex] \dfrac{dE}{dt} = \phi\Big(pqnE - cE\Big) \end{cases}$$

p= constant, q= constant, $\phi = 1$

Condition for fishery persistence



$pqK > c$



$pqK < c$

Mathematical Model of a Fishery with a Variable Price of the Resource

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Original Research Article

A model of a fishery with fish storage and variable price involving delay equations

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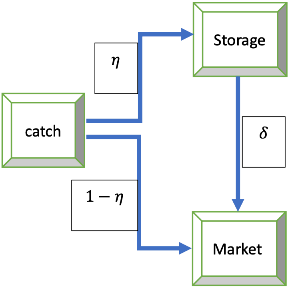
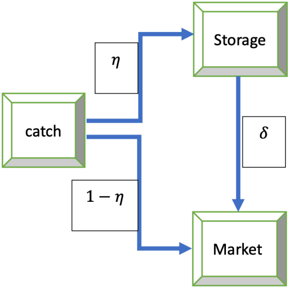


Mathematical Model of a Fishery with a Variable Price of the Resource

$$
\begin{cases}
\dfrac{dN}{d\tau} = \varepsilon[rN(1-N) - qNE] \\[2ex]
\dfrac{dE}{d\tau} = \varepsilon\Big[P[(1-\eta)qNE + \delta S] - cE\Big] \\[2ex]
\dfrac{dS}{d\tau} = \varepsilon[\eta qNE - \delta S] \\[2ex]
\dfrac{dP}{d\tau} = D(P) - [(1-\eta)qNE + \delta S]
\end{cases}
$$

$$D(P) = \frac{A}{P + p_0}$$

$$t = \epsilon\tau$$



Derivation of the aggregated model

$$\frac{dP}{d\tau} = \frac{A}{P + p_0} - [(1-\eta)qNE + \delta S] = 0 \tag{1}$$

This equation admits one solution P^* given by the following expression :

$$P^* = \frac{A}{(1-\eta)qNE + \delta S} - p_0 \tag{2}$$

The second equation now becomes

$$\frac{dE}{d\tau} = A - p_0[(1-\eta)qNE + \delta S] - cE$$

This aggregation makes us obtain the following system of three differential equations :

$$\begin{cases} \dot{N} = rN(1-N) - qNE \\ \\ \dot{E} = A - p_0[(1-\eta)qNE + \delta S] - cE \\ \\ \dot{S} = \eta qNE - \delta S \end{cases} \tag{3}$$

Derivation of the aggregated model

$$\begin{cases} \dot{N} = rN(1-N) - qNE \\ \\ \dot{E} = A - p_0 \left[(1-\eta)qNE + \eta q \displaystyle\int_{-\infty}^{0} \delta e^{\delta\theta} N_t(\theta) E_t(\theta) d\theta \right] - cE \end{cases} \tag{4}$$

where

$$N_t(\theta) = N(t+\theta), \quad E_t(\theta) = E(t+\theta) \text{ with } t \geq 0 \text{ and } \theta \leq 0$$

denote the history functions. In system (4), the kernel ω given by

$$\omega(\theta) = \delta e^{\delta\theta}, \ \theta \leq 0, \tag{5}$$

Qualitative analysis of the aggregated model

It can be checked that model (4) can admit up to three nonnegative equilibria : The "catastrophic" equilibrium defined by $CE = (0, A/c)$ and up to two nontrivial equilibria (N^*, E^*) where

$$E^* = \frac{A}{p_0 q N^* + c}$$

while $N^* \in (0, 1)$ is a positive root of the following quadratic equation :

$$F(N) := p_0 q N^2 + (c - p_0 q)N + \left(\frac{qA}{r} - c\right) = 0. \tag{6}$$

Qualitative analysis of the aggregated model

The number of non catastrophic equilibrium points of system (4) is given by the number of positive real roots of Eq.(6) on the interval $(0, 1)$.
As $F(1) = qA/r > 0$, we have the following two cases :
Case I : If $F(0) < 0$, that is if $c > qA/r$, then, there exists a unique positive equilibrium (N^*, E^*) where N^* is given by

$$N^* = \frac{-(c - p_0 q) + \sqrt{(c + p_0 q)^2 - 4\frac{p_0 q^2 A}{r}}}{2p_0 q}. \tag{7}$$

Case II : If $F(0) > 0$, that is if $c < qA/r$, we have the following two subcases :

• If $(c + p_0 q)^2 < \dfrac{4p_0 q^2 A}{r}$, the system (4) has no positive equilibrium.

• If $(c + p_0 q)^2 > \dfrac{4p_0 q^2 A}{r}$ and $c < p_0 q$, then there exist two positive equilibria $(N^\pm, E^\pm)$, where

$$N^\pm = \frac{-(c - p_0 q) \pm \sqrt{(c + p_0 q)^2 - 4\frac{p_0 q^2 A}{r}}}{2p_0 q} \quad \text{and} \quad E^\pm = \frac{A}{p_0 q N^\pm + c}. \tag{8}$$

We note that $0 < N^- < N^+ < 1$.

Stability Analysis

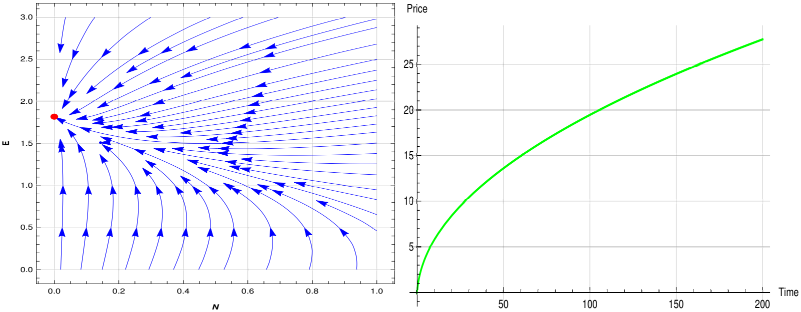


Figure – Dynamics of fish stock, fishing effort and price. **(Left)** Case of a stable CE with parameter values : $r = 1,\ q = 1,\ c = 2,\ p_0 = 1,\ A = 0.9,\ \delta = 1,\ \eta = 0.5$. Initial conditions are : $N(\theta) = 0.3,\ E(\theta) = 0.4$ for $-50 \leq \theta < 0$.**(Right)** Price versus time.

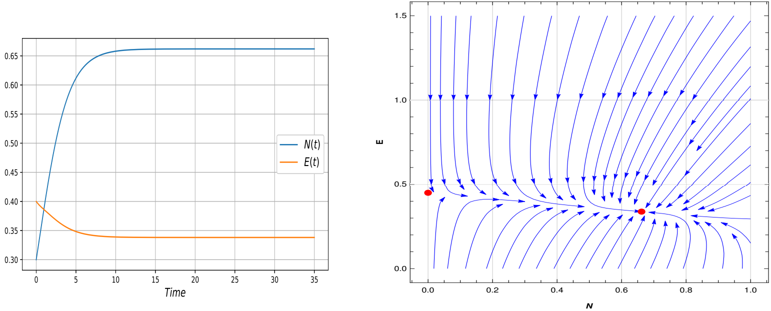


Figure – (**Left**) Dynamics of fish stock and fishing effort. (**Right**) The phase diagram obtained from the reduced model (3). Parameter values are :
$r = 1,\ q = 1,\ c = 2,\ p_0 = 1,\ A = 0.9,\ \delta = 1,\ \eta = 0.5$. Initial conditions are :
$N(\theta) = 0.3,\ E(\theta) = 0.4$ for $-50 \leq \theta \leq 0$.

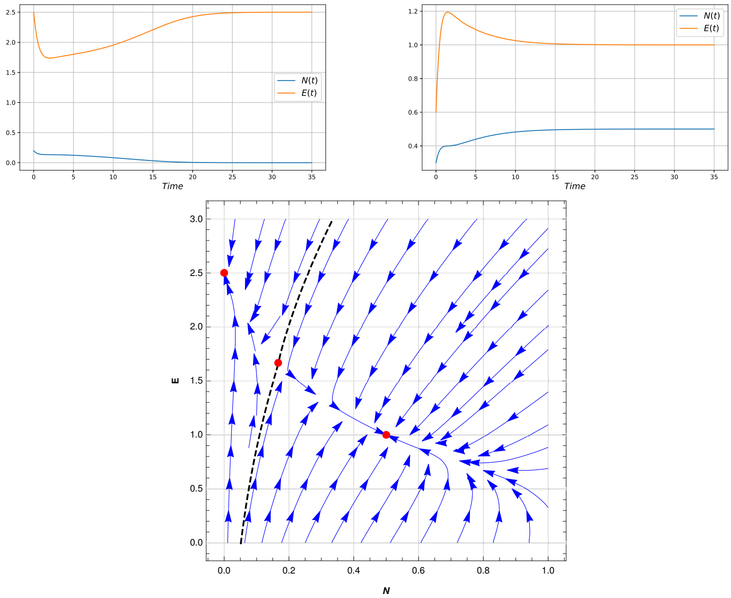


Figure – Dynamics of fish stock and fishing effort. Parameter values are :

Bifurcation diagram

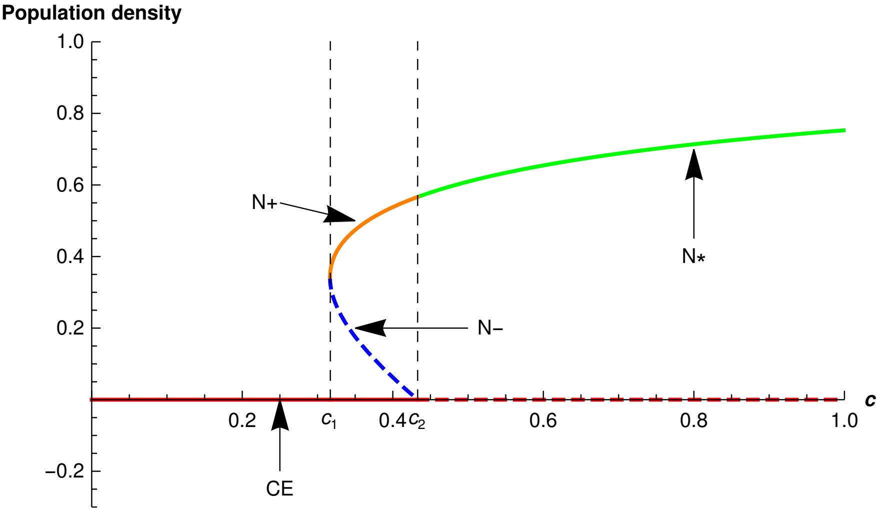


Figure – Bifurcation diagram in the plan (c, N). The other parameters are fixed as follows : $r = 3$, $p_0 = 2$, $A = 2.6$, $q = 0.5$.

Effect of fish storage on the dynamics of the fishery

We assume that the institution in charge of fisheries management decides to implement a tax per unit of fishing effort on the storage of fish.

We assume an additional storage cost $\alpha \frac{\eta}{\delta}$.

The total cost per unit of fishing effort reads as follows :

$$c = c_1 + \alpha \frac{\eta}{\delta} \tag{9}$$

where c_1 represents all the other costs such as fuel for the boats, fishermen's wages, other taxes, the minimum profit required by the shipowner, etc.

Effect of fish storage on the dynamics of the fishery

Recall that when $c > \dfrac{qA}{r}$, we have two equilibria $(0, \dfrac{A}{c})$ which is unstable and (N^*, E^*) is LAS and the previous condition rewrites :

$$\frac{\eta}{\delta} > \frac{1}{\alpha}(\frac{qA}{r} - c_1). \tag{10}$$

When $c < \dfrac{qA}{r}$ we have three equilibria $(0, \dfrac{A}{c})$, (N^-, E^-) and (N^+, E^+) where (N^-, E^-) is unstable and the two others equilibria are LAS. As we have $c = c_1 + \alpha\dfrac{\eta}{\delta}$, we have the inverse condition :

$$\frac{\eta}{\delta} < \frac{1}{\alpha}(\frac{qA}{r} - c_1) \tag{11}$$

Effect of fish storage on the dynamics of the fishery

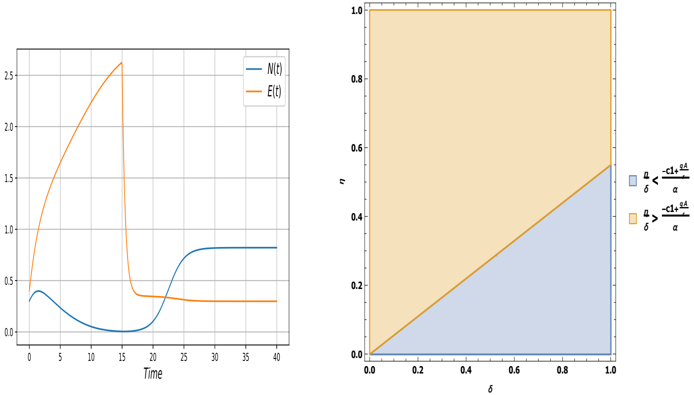
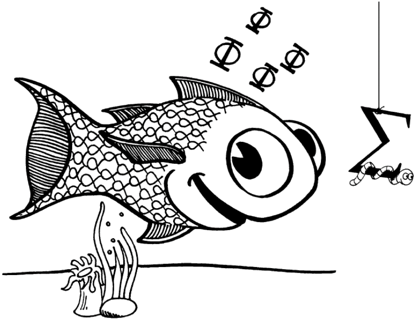
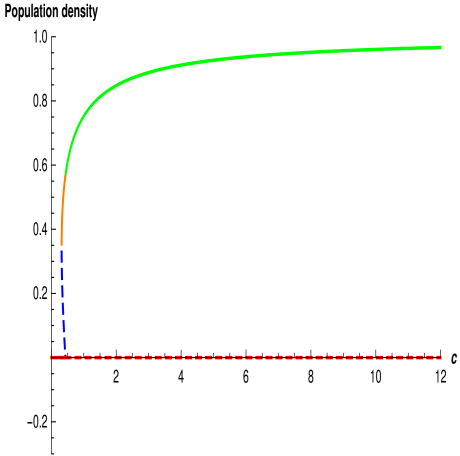


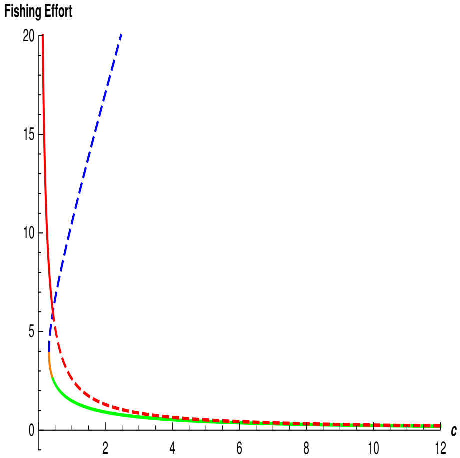
Figure – Graphic illustrations of the results, where storage parameters are changed from a certain time. The initial trajectory for $0 < t < 15$ moves towards the resource extinction equilibrium, then for $t > 15$, it is modified to move towards the sustainable fishery equilibrium. For $t \leq 15$, $\delta = 0.9$ and $\eta = 0.1$ and for $t > 15$ $\delta = 0.2$ and $\eta = 0.9$.

Thank you for your attention



Un poisson mathématique – par Theo Engell-Nielsen.


Population density
1.0
0.8
0.6
0.4
0.2
−0.2
2
4
6
8
10
12
c



Fishing Effort
20
15
10
5
0
2
4
6
8
10
12
c